Title: Basic Calculations with Logarithms
Class: Math 107 or Math 120
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Instructions to tutor: Read instructions under "Activity" and follow all steps for each problem exactly as given.
Keywords/Tags: logarithms, log functions, logarithmic functions

Objective: Understanding basic definition of logarithmic functions, and finding log values.

Activity: Do at least one example with a tutor first, then try the rest of the examples on your own. Take them to your tutor to make sure you're doing them correctly.

Introduction

Logarithmic functions are inverses of exponential functions. They are useful in many applications, especially in helping us solve exponential equations (which is covered in another DLA). Here, we will only discuss the basics.

To say $\log_b(a) = c$ means $b^c = a$. In both cases b is called the **base**, and we only talk about logarithms that have a positive base, not equal to one. $(b > 0, \quad b \neq 1)$

Keep this in mind: $\log_b(a) = c \quad \leftrightarrow \quad b^c = a$

So if we're trying to find the value of $\log_3 9$, we think of the question $\log_3 9 = ?$ and then rewrite this (using the above) as $3^? = 9$. So, what's the answer? 3 to what power is equal to 9? The answer is 2. Therefore:

$$\log_3 9 = 2$$

Using this same idea, and what you should already know about exponents, we can find basic log values.

Example 1. Fill in the blanks:

(a) $\log_5 25 =$ _____ because _____$^{\overline{\quad}}$ = _____

(b) $\log_7 7 =$ _____ because _____$^{\overline{\quad}}$ = _____

(c) $\log_3 27 =$ _____ because _____$^{\overline{\quad}}$ = _____

If all you have to do is find the value of a logarithm, you can put the question mark on top of the base:

To find the value of $\log_2 16$ think of $\log_{2^?} 16$. So, ask yourself, 2 to what power is 16? The answer is 4.

Therefore:
$$\log_2 16 = 4$$

Example 2. Find each logarithm.

(a) $\log_2(32)$ (b) $\log_5(25)$ (c) $\log_3(81)$ (d) $\log_4(64)$

A Special Logarithm

When the base is 10, we call the logarithm the **common logarithm**, and we don't write the base.

Example: $\log(100) = 2$ (because 10 to the second power equals 100)

Before you continue, make sure you review zero, negative, and fractional exponents. For example you need to know how to evaluate 9^0, 9^{-2}, $9^{\frac{1}{2}}$, $9^{\frac{3}{2}}$, etc.

Example 3. Find each logarithm.

(a) $\log_4\left(\dfrac{1}{16}\right)$

(b) $\log_{16}(4)$

(c) $\log_8(1)$

(d) $\log_6(6)$

(e) $\log_8(2)$

(f) $\log(10,000)$

(g) $\log(0.0001)$

(h) $\log_8\left(\log_6(36)\right)$ This is a "nested" logarithm. Evaluate the inside log first.

(i) $\log_b\left(\sqrt{b}\right)$

(j) $\log_b\left(\sqrt[3]{b^2}\right)$

(k) $\log_b\left(\log_b b\right)$

Example 4. Find each logarithm.

(a) $\log_8(0)$ Hmmmm…. must answer: "8 to what power equals 0?" Is there such a number??

(b) $\log_8(-8)$

Hopefully you were not able to find values for these! These expressions are undefined. This might help you understand why the domain of the function $y = \log_b(x)$ is the set of all **positive** real numbers. x cannot be 0 or any negative number.

Let's repeat: The domain of the function $y = \log_b(x)$ is _____

After you go over the previous problems with a tutor, try the following on your own, then check with a tutor to make sure you did them correctly.

For each of the following problems label the variables, set up a system, solve it and interpret the result.

1. Find each logarithm.

(a) $\log_3(9)$

(b) $\log_9(3)$

(c) $\log_3\left(\dfrac{1}{9}\right)$

(d) $\log(1)$

(e) $\log(10)$

(f) $\log(0.1)$

(g) $\log_3\left(\sqrt{3}\right)$

(h) $\log_5(5)$

(i) $\log_2\left(\dfrac{1}{8}\right)$

(j) $\log_b \sqrt[5]{b^3}$

(k) $\log_2\left(\log_b\left(\sqrt{b}\right)\right)$

(l) $\log_b(1)$

(m) $\log(-1)$

(n) $\log_b\left(\dfrac{1}{b^3}\right)$

(o) $\log_5(0)$

2. What is the domain of the function $y = \log_3(x)$?